On the sufficiency of backdoor-adjustment in DAGs

This is a short post highlighting that *backdoor* or covariate adjustment is always a valid strategy for estimating the causal effect of a single treatment on a single outcome in causal models of a DAG with no unmeasured confounders. In particular, the parents of the treatment variable serve as a sufficient adjustment set that blocks all backdoor paths. This may seem obvious to veterans of causal inference but it was quite surprising to me when I first encountered this fact.

In the following, I will just show that the counterfactual mean $\mathbb{E}[Y(t)]$ (the expected value of the outcome Y had treatment been assigned to $T = t$) is identified via backdoor-adjustment. A contrast between two different assignments is easy to compute via $\mathbb{E}[Y(t)] - \mathbb{E}[Y(t')]$ as the identification argument for different treatment assignments is exactly the same.

Lemma 1. *Given a distribution $p(V)$ that is Markov relative to a DAG $\mathcal{G}$, the causal effect of a treatment variable T on an outcome variable Y (wlog Y is a descendant of T) is given by the backdoor-adjustment formula*

$$\mathbb{E}[Y(t)] = \mathbb{E}\big[\mathbb{E}[Y \mid T = t, \mathrm{pa}_{\mathcal{G}}(T)]\big],$$

where $\mathrm{pa}_{\mathcal{G}}(T)$ are the parents of T in $\mathcal{G}$.

Proof. The proof is quite simple. First imagine a graph $\mathcal{G}'$ that is exactly the same as $\mathcal{G}$ but we delete all paths of the form $T \rightarrow \cdots \rightarrow Y$. That is, all directed (causal) paths from T to Y are absent in $\mathcal{G}'$ and all that is left are backdoor paths $T \leftarrow \cdots \rightarrow Y$ and paths of the form $T \cdots \rightarrow X \leftarrow \cdots Y$ that are blocked due to the presence of colliders. Thus, if we can show that the set $\mathrm{pa}_{\mathcal{G}}(T)$ d-separates T and Y in $\mathcal{G}'$, we have shown that all backdoor paths between T and Y are blocked. One easy argument for this is via the local Markov property of DAGs – each vertex is independent of its non-descendants (non-parents) given its parents. By construction of $\mathcal{G}'$, the outcome Y is a non-descendant of T and hence $T \perp\!\!\!\perp Y \mid \mathrm{pa}_{\mathcal{G}}(T)$. Alternatively, one can apply the d-separation criterion directly and see that we are not conditioning on any vertex that is a collider or a descendant of a collider on a path between T and Y and further, all paths of the form $T \leftarrow X \cdots \rightarrow Y$ are blocked as $X \in \mathrm{pa}_{\mathcal{G}}(T)$.

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